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Non-Supersymmetric Charged Domain Walls

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Abstract

We present general non-supersymmtric domain wall solutions with non-trivial scalar and gauge fields for gauged five-dimensional $N = 2$ supergravity coupled to abelian vector multiplets.

1 Introduction

The recent interest in the study of solutions of gauged supergravity theories in various dimensions has to a large extent been motivated by the conjectured Anti-de Sitter/Conformal Field Theory (AdS/CFT) equivalence [1]. From the CFT perspective, supergravity vacua could correspond to an expansion around non-zero vacuum expectation values of certain operators, or describe a holographic renormalization group flow [2]. It is hoped that this conjectured equivalence can help in gaining some understanding of the nonperturbative structure of gauge theories by studying classical supergravity solutions. Of particular interest are the domain walls of gauged five dimensional $N = 2$ supergravity theories. The theory of ungauged five-dimensional $N = 2$ supergravity coupled to abelian vector supermultiplets can be obtained by compactifying eleven-dimensional supergravity, the low-energy limit of M-theory, on a Calabi-Yau three-fold [3]. Another class of models are those obtained in [4] which are closely related to Jordan algebras. The gauged five-dimensional $N = 2$ supergravity theories we consider are those obtained by gauging the $U(1)$ subgroup of the $SU(2)$ automorphism group of the superalgebra [4]. The gauging is accomplished by introducing into the Lagrangian of the theory a linear combination of the abelian vector fields already present in the ungauged theory, i. e. $A_\mu = V_I A^I_\mu$, with a coupling constant χ. The coupling of the fermions of the theory to the $U(1)$ vector field breaks supersymmetry and gauge-invariant terms are added to preserve $N = 2$ supersymmetry. In terms of the bosonic action of the theory, we get an additional χ^2-dependent scalar potential $\mathcal{V}$ [4].

Most domain wall solutions constructed so far are configurations preserving some of the supersymmetries (see for example [5]). Explicit supersymmetric domain wall solutions for the theories of [4], where the scalar fields live on symmetric spaces, were given in [6]. These solutions, describing a holographic renormalization group flow, were expressed in terms of Weierstrass elliptic function. Recently a systematic approach has been employed in the classification of general supersymmetric solutions of the gauged five dimensional supergravity with non-trivial vector multiplets [7, 8]. In [8], the requirement that the scalar manifold is a symmetric space was relaxed and the structure of solutions with null Killing vector in both gauged and ungauged supergravity theories was also investigated. In our present work we are interested in finding non-supersymmetric domain wall solutions. We present non-supersymmetric charged domain wall solutions with non-trivial scalars for all gauged five-dimensional $N = 2$ supergravity models coupled to vector multiplets. We organize our work as follows. In section two, and in an attempt to make our work self-contained, a brief review of the theories of the $U(1)$-gauged supergravity and their equations of motion are given. In section three, the domain wall solutions of [6] are presented as well as general domain wall solutions applicable for a Calabi-Yau compactification [8]. The analysis and the derivation of non-supersymmetric charged domain wall solutions are given in section four. Section five includes the study of the causal structure of the domain walls geometry and we conclude in section six.

2 Gauged Five-Dimensional N=2 Supergravity

The bosonic action of the gauged $D = 5$ $N = 2$ supergravity can be written as [4]

$$S = \frac{1}{16\pi G} \int \left(R + 2\chi^2 \mathcal{V} - G_{IJ} F^I \wedge *F^J - G_{IJ} dX^I \wedge \star dX^J - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K \right), \tag{2.1}$$

where I, J, K take values $1, \ldots, n$ and $F^I = dA^I$. Here the X^I represent the scalar fields of the theory, which are constrained via

$$\frac{1}{6} C_{IJK} X^I X^J X^K = 1,$$

and may be regarded as being functions of $n - 1$ unconstrained scalars ϕ^a. In addition, the couplings G_{IJ} depend on the scalars via

$$G_{IJ} = \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K \tag{2.2}$$

where $X_I \equiv \frac{1}{6} C_{IJK} X^J X^K$ and therefore one has the following useful relations

$$G_{IJ} X^J = \frac{3}{2} X_I \,, \qquad G_{IJ} \partial_a X^J = -\frac{3}{2} \partial_a X_I \,. \tag{2.3}$$

The scalar potential of the gauged theory can be written in the form

$$\mathcal{V} = 9 V_I V_J (X^I X^J - \frac{1}{2} G^{IJ}) \tag{2.4}$$

where V_I are constants [4]. The Einstein equations derived from the action (2.1) are given by

$$R_{\mu\nu} = G_{IJ} \left(F^I{}_{\mu\lambda} F^J{}_\nu{}^\lambda - \frac{1}{6} g_{\mu\nu} F^I{}_{\rho\sigma} F^{J\rho\sigma} + \nabla_\mu X^I \nabla_\nu X^J \right) - \frac{2}{3} \chi^2 \mathcal{V} \, g_{\mu\nu}. \tag{2.5}$$

The Maxwell equations are

$$d \left(G_{IJ} \star F^J \right) + \frac{1}{4} C_{IJK} F^J \wedge F^K = 0 \,. \tag{2.6}$$

The scalar equations of motion give the following relations [7, 8]

$$\begin{aligned} & d\left(\star dX_I\right) - \left(\frac{1}{6} C_{MNI} - \frac{1}{2} X_I C_{MNJ} X^J \right) dX^M \wedge \star dX^N \\ & + \left(X_M X^P C_{NPI} - \frac{1}{6} C_{MNI} - 6 X_I X_M X_N + \frac{1}{6} X_I C_{MNJ} X^J \right) F^M \wedge \star F^N \\ & + 3\chi^2 \left(\frac{1}{2} V_M V_N G^{ML} G^{NP} C_{LPI} + X_I G^{MN} V_M V_N - 2 X_I X^M V_M X^N V_N \right) \mathrm{dvol} = 0 \,. \end{aligned} \tag{2.7}$$

3 Supersymmetric Domain Walls

In this section we review the supersymmetric domain wall solutions found in [6] as well as the general supersymmetric solutions with null Killing vector and with vanishing gauge fields presented in [8]. In [6] one starts with the following general ansatz for supersymmetric domain wall solutions:

$$ds^2 = e^{2A}(-dt^2 + dz^2) + e^{2B}(dr^2 + r^2 d\theta^2 + r^2 d\phi^2), \tag{3.1}$$

where A and B are functions of the radial coordinate r only. The analysis of the Killing spinor equations (obtained from the vanishing of the gravitini and dilatino supersymmetric variations) gives the following restrictions on the metric and the scalar fields,

$$\begin{aligned} \partial_r A - \partial_r B - \frac{1}{r} &= 0\,, \\ \frac{1}{2}e^{-B}\partial_r X_I + \chi V_I + e^{-B} X_I(\partial_r W + \frac{1}{r}) &= 0\,. \end{aligned} \tag{3.2}$$

Eqn. (3.2) implies

$$X_I = \frac{1}{r^2}e^{-2B}[-2\chi V_I \int e^{3B} r^2 dr + \Lambda_I]\,, \tag{3.3}$$

where the Λ_I are integration constants. For symmetric spaces where

$$\begin{aligned} C^{IJK} &= \delta^{II'}\delta^{JJ'}\delta^{KK'} C_{I'J'K'}\,, \\ C_{IJK}C_{J'(LM}C_{PQ)K'}\delta^{JJ'}\delta^{KK'} &= \frac{4}{3}\delta_{I(L}C_{MPQ)}, \\ \mathcal{V} &= 27 C^{IJK} V_I V_J X_K\,, \end{aligned} \tag{3.4}$$

it was shown that the above equations are completely integrable. Defining the quantity [6]

$$y(u) = -9a \int e^{3(B+u)} du + \frac{9}{2}\chi^2 b\,, \tag{3.5}$$

with

$$a = C^{IJK} V_I V_J V_K\,, \qquad b = C^{IJK} V_I V_J \Lambda_K\,, \qquad c = C^{IJK} V_I \Lambda_J \Lambda_K\,, \qquad d = C^{IJK} \Lambda_I \Lambda_J \Lambda_K\,, \tag{3.6}$$

and where the new radial coordinate u is given by $u = \ln \chi r$, we obtain the differential equation

$$\left(\frac{dy}{du}\right)^2 = 4y^3 - g_2 y - g_3\,, \tag{3.7}$$

where

$$g_2 = 243\chi^4(b^2 - ac)\,, \qquad g_3 = \frac{729}{2}\chi^6(3abc - a^2d - 2b^3)\,. \tag{3.8}$$

The general solution of Eqn. (3.7) is given by $y = \wp(u+\gamma)$,where $\wp(u)$ denotes the Weierstrass elliptic function, and γ is an integration constant.

In the classification of solutions with null Killing vector and vanishing gauge field strengths in gauged supergravity [8], it was found that the metric and the scalar fields can be written in the following form

$$\begin{aligned} ds^2 &= H^{-1}\left(2dU(dV + \frac{1}{2}\mathcal{F}dU) - (dx^2)^2 - (dx^3)^2\right) - H^2(dx^1)^2\ , \\ H^{-1}X_I &= -2\chi V_I x^1 + \beta_I(U), \end{aligned} \tag{3.9}$$

with $\mathcal{F}$ given by

$$H\partial_1^2\mathcal{F} + H^4(\partial_2^2 + \partial_3^2)\mathcal{F} - 3\partial_1 H\partial_1\mathcal{F} = \frac{9}{2}H^6G^{IJ}\partial_U\beta_I\partial_U\beta_J\ . \tag{3.10}$$

Hence we see that solutions for which $\mathcal{F} = 0$ must have $\partial_U\beta_I = 0$ and hence H and X^I are also independent of U.

Changing to signature $(-,+,+,+,+)$ and concentrating on solutions with $\mathcal{F} = 0$, we obtain the following domain wall solutions

$$ds^2 = H^{-1}\left(-dt^2 + d\omega^2 + dx^2 + dy^2\right) + H^2dz^2\ , \tag{3.11}$$

$$H^{-1}X_I = -2\chi V_I z + \beta_I. \tag{3.12}$$

Notice that these solutions are valid for all gauged $N=2$ supergravity theories and in particular for those obtained from a Calabi-Yau compactification. To recover the domain wall solution of [6] described above, one can perform the following change of variable

$$H^3\left(\frac{dz}{du}\right)^2 = 1, \quad z = \frac{b}{2\chi a} - \frac{y}{9a\chi^3}, \tag{3.13}$$

4 Non-Supersymmetric Domain Walls

In this section we consider non-supersymmetric domain wall solutions which in certain limits give the supersymmetric solutions considered in the previous section. As an ansatz for non-supersymmetric solution we take:

$$ds^2 = \frac{1}{H}\left(-fdt^2 + dw^2 + dx^2 + dy^2\right) + \frac{H^2}{f}dz^2 \tag{4.1}$$

where f and H are functions of z only. The non-vanishing components of the Ricci tensor are given by

$$
\begin{aligned}
R_{tt} &= \frac{f}{2H^5}\left(4fH'^2 - fH''H + f''H^2 - 4HH'f'\right), \\
R_{xx} = R_{yy} = R_{ww} &= \frac{1}{2H^5}\left(-4fH'^2 + fHH'' + HH'f'\right), \\
R_{zz} &= \frac{2H''}{H} - \frac{5H'^2}{H^2} + \frac{2H'f'}{fH} - \frac{f''}{2f},
\end{aligned} \tag{4.2}
$$

where the prime denotes differentiation with respect to the coordinate z. The Einstein equations of motion (2.5) give the following conditions:

$$
G_{IJ}F^I_{zt}F^J_{zt} = \frac{1}{2H^3}\left(f''H^2 - 3H'f'H\right), \tag{4.3}
$$

$$
G_{IJ}\partial_z X^I \partial_z X^J = \frac{3}{2H^2}\left(H''H - 2H'^2\right), \tag{4.4}
$$

$$
\chi^2 \mathcal{V} = \frac{3f}{4H^4}\left(4H'^2 - HH''\right) + \frac{1}{4H^3}\left(Hf'' - 6H'f'\right) \tag{4.5}
$$

where we allowed the gauge fields to have non-vanishing field strengths F^I_{zt}. Note that the function f drops out in (4.4) and as a consequence we will not modify the scalars and we will simply use the ansatz as given in the supersymmetric case (3.12). Then it can be easily demonstrated that

$$
V_I X^I = \frac{1}{2\chi}\frac{H'}{H^2}, \tag{4.6}
$$

$$
G^{IJ}V_I V_J = \frac{1}{6\chi^2}\left(\frac{H''}{H^3} - \frac{H'^2}{H^4}\right). \tag{4.7}
$$

Thus the scalar potential is given by

$$
\mathcal{V} = \frac{3}{4\chi^2 H^4}\left(4H'^2 - HH''\right). \tag{4.8}
$$

Upon comparing (4.8) with the expression of $\mathcal{V}$ in (4.5), the following condition is obtained

$$
6HH'f' - 3(1-f)\left(HH'' - 4H'^2\right) - f''H^2 = 0. \tag{4.9}
$$

This can be solved by

$$
f = 1 + (\mu + \alpha z)H^3 \tag{4.10}
$$

where μ and α are constants. Going back to the gauge equation of motion (2.6), this gives for our solution

$$
\partial_z\left(\frac{1}{H^2}G_{IJ}F^J_{zt}\right) = 0, \tag{4.11}
$$

from which we obtain

$$F^I_{zt} = H^2 G^{IJ} q_J. \tag{4.12}$$

where q_I are constants representing electric charges. Using (4.3), (4.10) and (4.12), we get

$$G^{IJ} q_I q_J = \frac{3}{2H^4} \left[\alpha H H' + (\mu + \alpha z)\left(H H'' - H'^2\right)\right]. \tag{4.13}$$

Let us first consider the case with vanishing charges $q_I = 0$, and take $\alpha \neq 0$. In this case, one solution of (4.13) is given by

$$H = \frac{c}{(\mu + \alpha z)} \ . \tag{4.14}$$

Then (4.4) and (3.12) imply that the scalars are constants, with $X_I = -2\frac{\chi c}{\alpha} V_I$.

If however, one takes $q_I \neq 0$, with $\alpha = 0$ in (4.13) then using (4.7) we obtain the condition

$$G^{IJ}\left(q_I q_J - 9\mu\chi^2 V_I V_J\right) = 0. \tag{4.15}$$

This can be solved by

$$q_I = 3\sqrt{\mu}\chi V_I. \tag{4.16}$$

Finally it remains to check whether the scalar equations of motion are satisfied for our solution. The scalar equations (2.7) for our solution give

$$\begin{aligned}
& H\partial_z\left(f H^{-3}\partial_z X_I\right) - f H^{-2}\left(\frac{1}{6}C_{MNI} - \frac{1}{2}X_I C_{MNJ}X^J\right)\partial_z X^M \partial_z X^N \\
& - H^{-1}\left(X_M X^P C_{NPI} - \frac{1}{6}C_{MNI} - 6X_I X_M X_N + \frac{1}{6}X_I C_{MNJ}X^J\right) F^M_{tz} F^N_{tz} \\
& + 3\chi^2\left(\frac{1}{2}V_M V_N G^{ML} G^{NP} C_{LPI} + X_I G^{MN} V_M V_N - 2X_I X^M X^N V_M V_N\right) = 0 \ .
\end{aligned} \tag{4.17}$$

To simplify the calculation, we multiply the scalar equations for the supersymmetric case, i. e. multiply

$$\begin{aligned}
& H\partial_z\left(H^{-3}\partial X_I\right) - H^{-2}\left(\frac{1}{6}C_{MNI} - \frac{1}{2}X_I C_{MNJ}X^J\right)\partial_z X^M \partial_z X^N \\
& + 3\chi^2\left(\frac{1}{2}V_M V_N G^{ML} G^{NP} C_{LPI} + X_I G^{MN} V_M V_N - 2X_I X^M X^N V_M V_N\right) = 0 \ .
\end{aligned} \tag{4.18}$$

with f and subtract the resulting equation from (4.17), this gives after using the solution for the gauge fields,

$$H'\partial_z X_I + \chi^2\left(4X^K V_I V_K - 4X^K X^L X_I V_L V_K\right) H^3 = 0 \ . \tag{4.19}$$

It can be easily seen that this equation is indeed satisfied for our solution.

To summarize, we have obtained a class of domain wall solutions for all gauged five-dimensional $N = 2$ supergravity theories coupled to an arbitrary number of vector multiplets. These solutions are given by

$$\begin{aligned} ds^2 &= -\frac{(1+\mu H^3)}{H}dt^2 + \frac{1}{H}\left(dw^2 + dx^2 + dy^2\right) + \frac{H^2}{1+\mu H^3}(dz)^2, \\ F^J_{zt} &= 3H^2 G^{IJ}\sqrt{\mu}V_I\chi, \\ X_I &= H\left(-2\chi V_I z + \beta_I\right). \end{aligned} \tag{4.20}$$

In general, the metric is specified only implicitly by (4.20), because H is not specified explicitly by the equation of X_I. However, when the scalar manifold is symmetric, we have the relation

$$\frac{9}{2}C^{IJK}X_I X_J X_K = 1 \tag{4.21}$$

from which we can explicitly solve for H and find

$$H = (\alpha_0 + \alpha_1 z + \alpha_2 z^2 + \alpha_3 z^3)^{-\frac{1}{3}} \tag{4.22}$$

where

$$\begin{aligned} \alpha_0 &= \frac{9}{2}C^{IJK}\beta_I\beta_J\beta_K, \\ \alpha_1 &= -27\chi C^{IJK}V_I\beta_J\beta_K, \\ \alpha_2 &= 54\chi^2 C^{IJK}V_I V_J\beta_K, \\ \alpha_3 &= -36\chi^3 C^{IJK}V_I V_J V_K. \end{aligned} \tag{4.23}$$

For the special case of the STU model solutions, for which the intersection numbers are given by

$$C_{IJK} = |\epsilon_{IJK}| \tag{4.24}$$

for $I, J, K = 1, 2, 3$. In this case, H factorizes as

$$H = (\beta_0 z + \lambda_0)^{-\frac{1}{3}}(\beta_1 z + \lambda_1)^{-\frac{1}{3}}(\beta_2 z + \lambda_2)^{-\frac{1}{3}} \tag{4.25}$$

for constants $\beta_0, \beta_1, \beta_2, \lambda_0, \lambda_1, \lambda_2$. Note that as $z \to -\frac{\lambda_i}{\beta_i}$, then the Ricci scalar diverges as $(\beta_i z + \lambda_i)^{-\frac{4}{3}}$ if $\mu = 0$, and as $(\beta_i z + \lambda_i)^{-\frac{7}{3}}$ if $\mu > 0$.

Hence we observe that both the supersymmetric and non-supersymmetric domain wall solutions contain curvature singularities. However the causal structure of the spacetimes differs considerably between the supersymmetric and non-supersymmetric cases.

5 Causal Structure of Domain Wall Spacetime

To proceed, we examine the causal structure of the spacetime geometry given in (4.20) for the solutions with symmetric scalar manifolds. Observe that geodesics on the spacetime with metric (4.20) have the following conserved quantities

$$E = \frac{1}{H}(1 + \mu H^3)\dot{t} \tag{5.1}$$

$$P^i = \frac{1}{H}\dot{x}^i \tag{5.2}$$

for $i = 1, 2, 3$, where $t = t(\tau)$, $(x^1, x^2, x^3) = (x(\tau), y(\tau), w(\tau))$, $z = z(\tau)$, $\dot{} = \frac{d}{d\tau}$ and τ is an affine parameter. We will restrict our consideration to geodesic motion in the domain of z for which $H > 0$, and we take $\mu > 0$. It is convenient to define $P^2 = (P^1)^2+(P^2)^2+(P^3)^2$. Then null geodesics satisfy

$$\left(\frac{dz}{d\tau}\right)^2 = \frac{1}{H}\left(E^2 - (1 + \mu H^3)P^2\right) \tag{5.3}$$

whereas timelike geodesics satisfy

$$\left(\frac{dz}{d\tau}\right)^2 = \frac{1}{H}E^2 - \frac{1}{H^2}(1 + \mu H^3)\left(HP^2 + 1\right) \ . \tag{5.4}$$

Note that as $z \rightarrow \infty$, $H \sim z^{-1}$, and hence (5.4) implies that no timelike geodesic of fixed E, P^i can reach $z = \pm\infty$. In addition, causal geodesics must satisfy

$$E^2 - P^2 > \frac{E^2}{1 + \mu H^3} - P^2 \geq \frac{H}{1 + \mu H^3}\left(\frac{dz}{d\tau}\right)^2 \geq 0$$

which implies that $E^2 > P^2$.

Null geodesics of the supersymmetric solution satisfy

$$\left(\frac{dz}{d\tau}\right)^2 = \frac{1}{H}(E^2 - P^2) \ . \tag{5.5}$$

In the neighborhood of one of the curvature singularities, one can take $H \sim \alpha z^{-\frac{1}{3}}$ as $z \rightarrow 0$. It follows that a null geodesic reaches the curvature singularity within finite affine parameter. Also, null geodesics can propagate out to $z = \infty$, though they do not reach $z = \infty$ in finite affine parameter.

Timelike geodesics of the supersymmetric solution satisfy

$$\left(\frac{dz}{d\tau}\right)^2 = \frac{1}{H}(E^2 - P^2) - \frac{1}{H^2} \ . \tag{5.6}$$

Again, timelike geodesics reach the curvature singularity in finite proper time, but are confined to lie within $0 \leq z \leq z_{max}(E, P^2)$.

Null geodesics of the non-supersymmetric solution with $P^2 \neq 0$ cannot reach the singularity. However, null geodesics with $P^2 = 0$ satisfy

$$\left(\frac{dz}{d\tau}\right)^2 = \frac{1}{H}E^2 \tag{5.7}$$

and reach the singularity in finite affine parameter. In both cases, the null geodesics can propagate out to $z = \infty$, though they do not reach $z = \infty$ in finite affine parameter.

Timelike geodesics of the non-supersymmetric solution cannot reach the singularity for any choice of P, and are therefore confined within region $0 < z_{min}(E, P^2) \leq z \leq z_{max}(E, P^2)$.

6 Conclusion

In this paper we have constructed non-supersymmetric domain wall solutions of gauged five dimensional $N = 2$ supergravity theories with non trivial vector multiplets. The causal structure of these solutions was also discussed. These solutions constitute generalizations to a subclass of null solutions with vanishing gauge fields which were considered in [8]. In the supersymmetric limit the scalar fields remain unchanged and the gauge field strengths vanish. The scalar fields structure of these domain wall solutions resembles those for black hole solutions considered in [9] and therefore explicit domain wall solutions for the Calabi-Yau models considered in [9] can be constructed.

It will be of interest to find non-supersymmetric generalizations to the solutions of [7,8] and in particular to the supersymmetric null solutions with non-trivial gauge fields of [8]. We hope to report on this in a future publication.

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